On the Minimum Induced Drag of Wings

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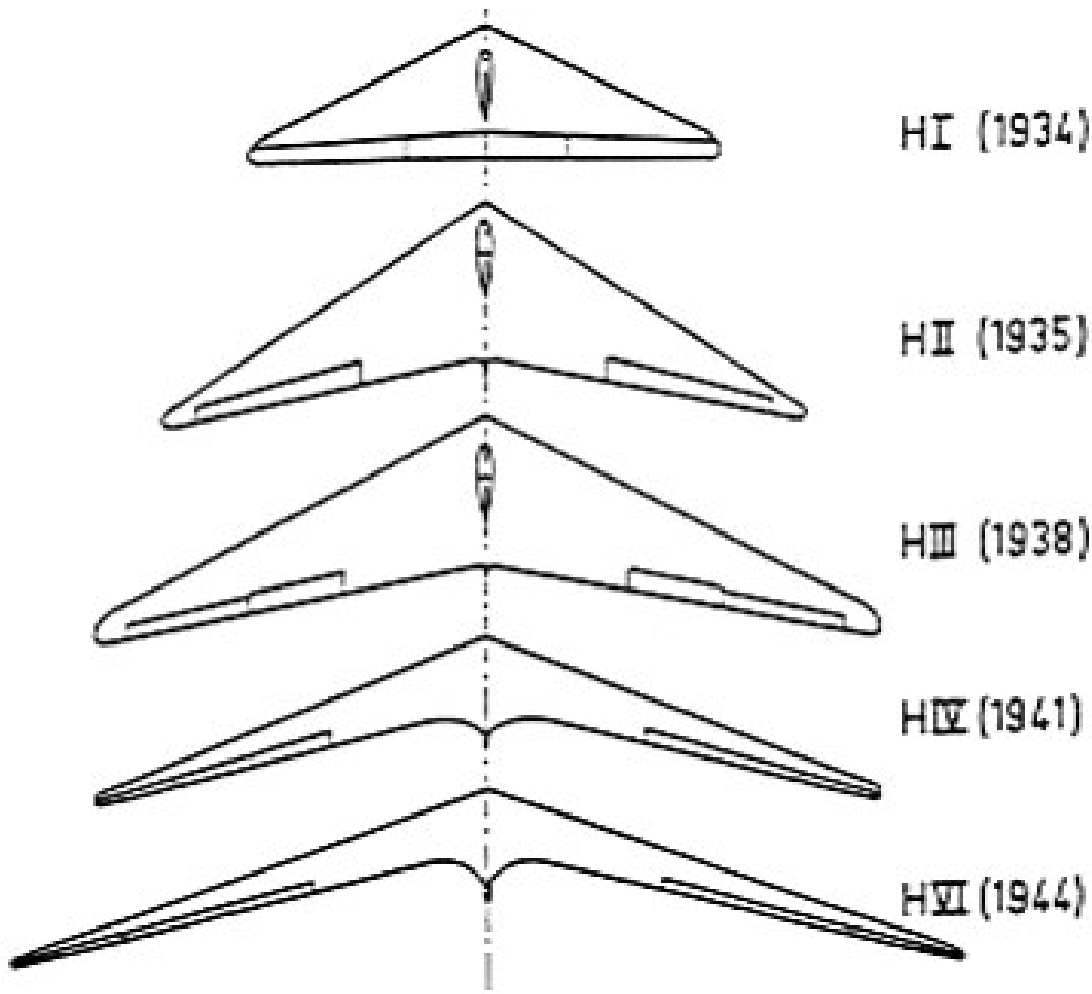
Introduction

- The History of Spanload
  Development of the optimum spanload
  Winglets and their implications
- Horten Sailplanes
- Flight Mechanics & Adverse yaw
- Concluding Remarks

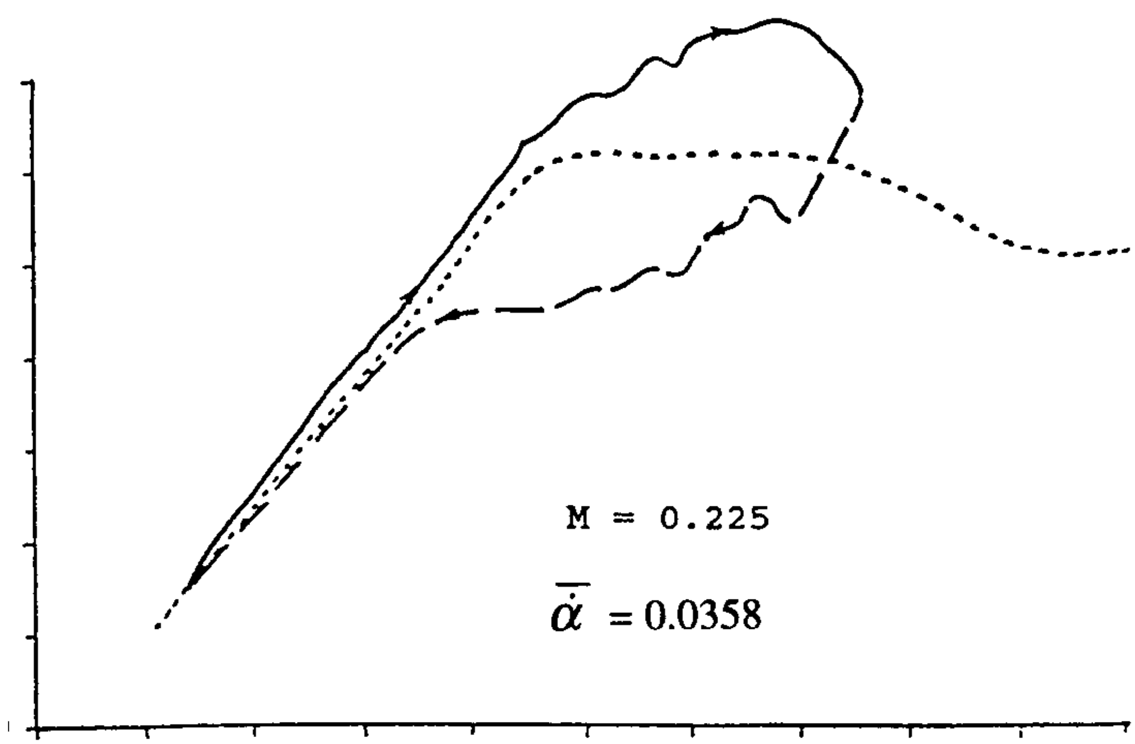
History

- Bird Flight as the Model for Flight
- Vortex Model of Lifting Surfaces
- Optimization of Spanload
  Prandtl
  Prandtl/Horten/Jones
  Klein/Viswanathan
- Winglets - Whitcomb



Bird Flight as a Model

- Propulsion
  Flapping motion to produce thrust
  Wings also provide lift
  Dynamic lift - birds use this all the time (easy for them, hard for us)
- Stability and Control
  Still not understood in literature
  Lack of vertical surfaces
- Birds as an Integrated System
  Structure
  Propulsion
  Lift (performance)
  Stability and control



Spanload Development

- Ludwig Prandtl
  Development of the boundary layer concept (1903)
  Developed the “lifting line” theory
  Developed the concept of induced drag
  Calculated the spanload for minimum induced drag (1908?)
  Published in open literature (1920)
- Albert Betz
  Published calculation of induced drag
  Published optimum spanload for minimum induced drag (1914)
  Credited all to Prandtl (circa 1908)
- Max Munk
  General solution to multiple airfoils
  Referred to as the “stagger biplane theorem” (1920)
  Munk worked for NACA Langley from 1920 through 1926
- Prandtl (again!)
  “The Minimum Induced Drag of Wings” (1932)
  Introduction of new constraint to spanload
  Considers the bending moment as well as the lift and induced drag

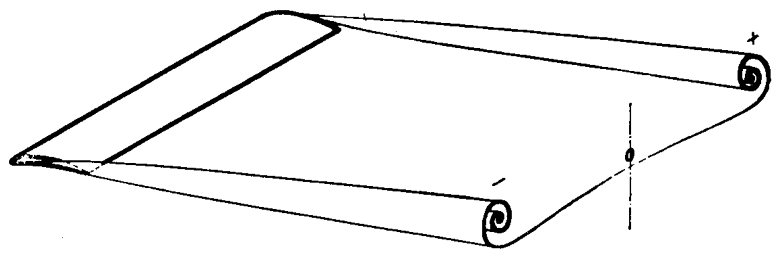
Practical Spanload Developments

- Reimar Horten (1945)
  Use of Prandtl's latest spanload work in sailplanes & aircraft
  Discovery of induced thrust at wingtips
  Discovery of flight mechanics implications
  Use of the term "bell shaped" spanload

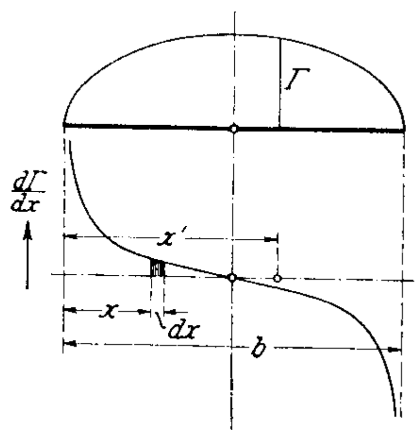
- Robert T Jones
  Spanload for minimum induced drag and wing root bending moment
  Application of wing root bending moment is less general than Prandtl's
  No prior knowledge of Prandtl's work, entirely independent (1950)

- Armin Klein & Sathy Viswanathan
  Minimum induced drag for given structural weight (1975)
  Includes bending moment
  Includes shear

Prandtl Lifting Line Theory



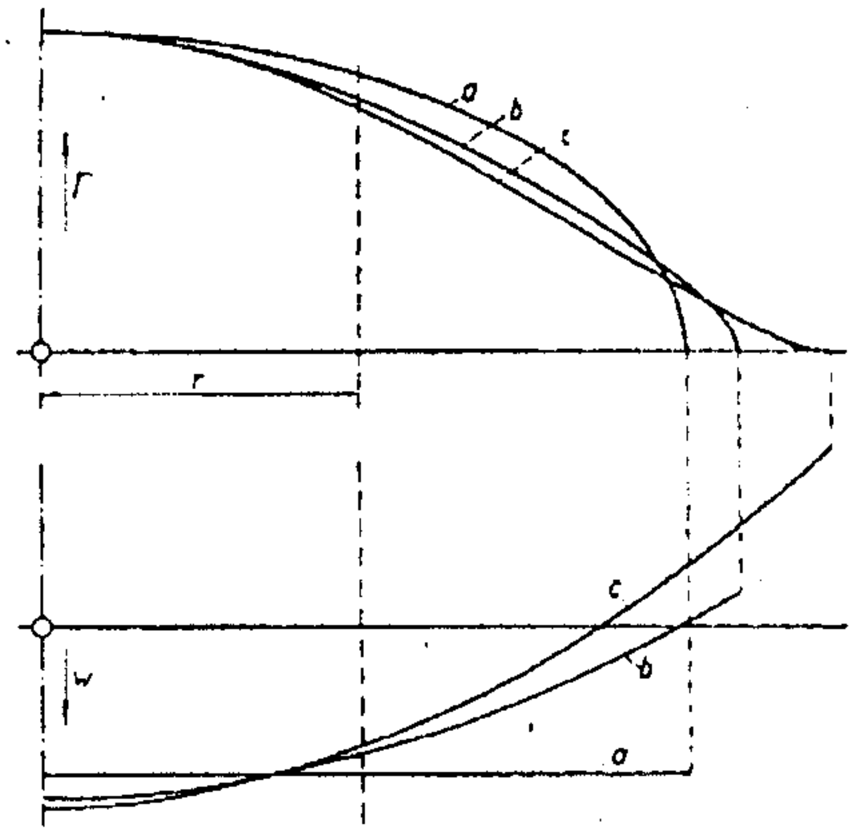
- Prandtl's "vortex ribbons"



- Elliptical spanload (1914)

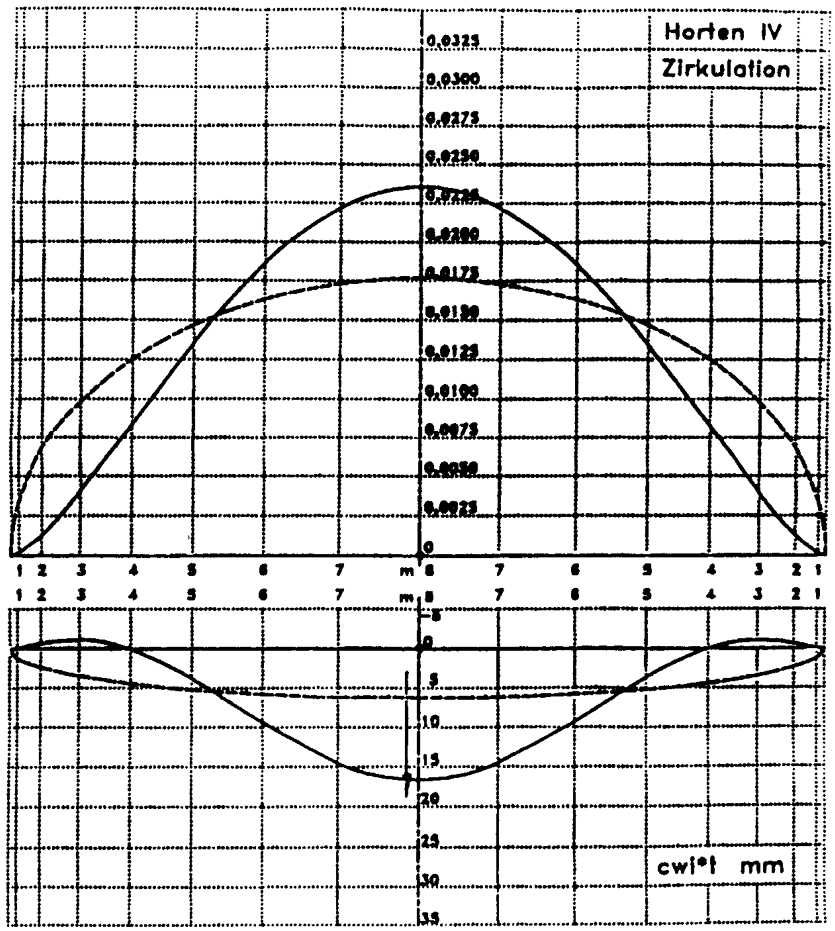
- "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift." y = c

Minimum Induced Drag & Bending



- Prandtl (1932)
  Constrain minimum induced drag
  Constrain bending moment
  22% increase in span with 11% decrease in induced drag

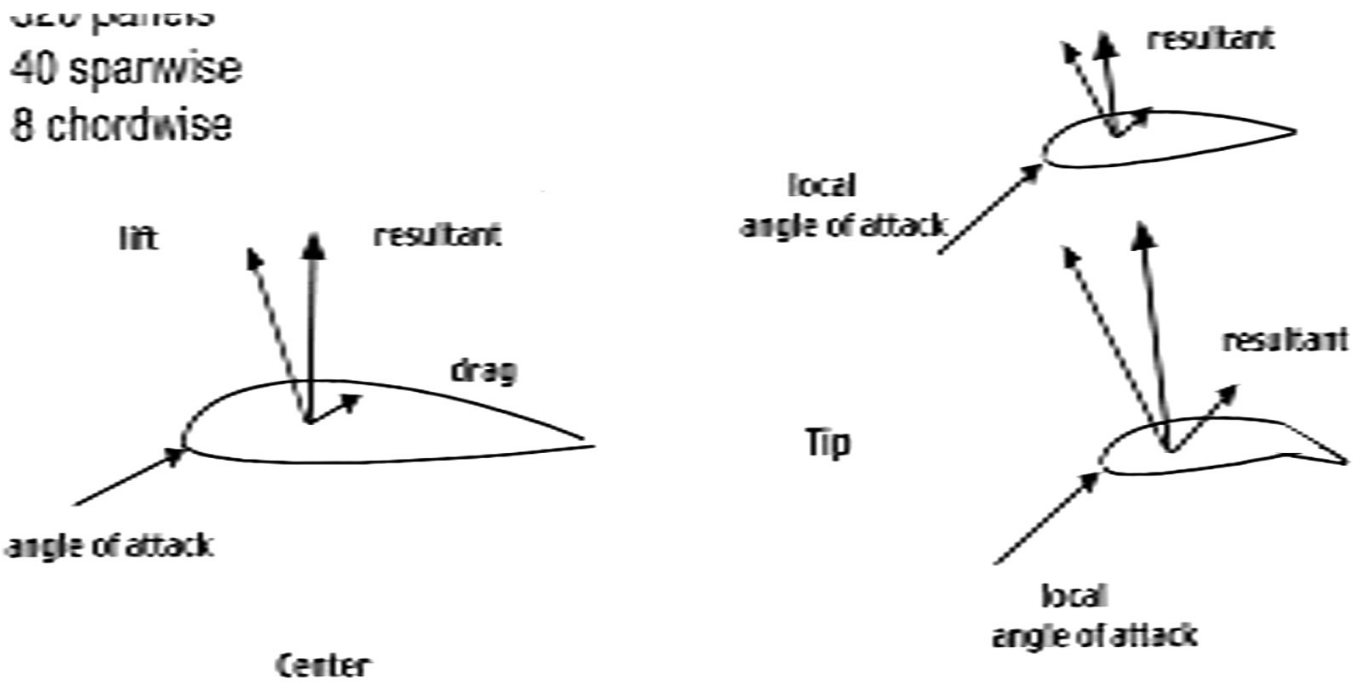
Horten Applies Prandtl's Theory



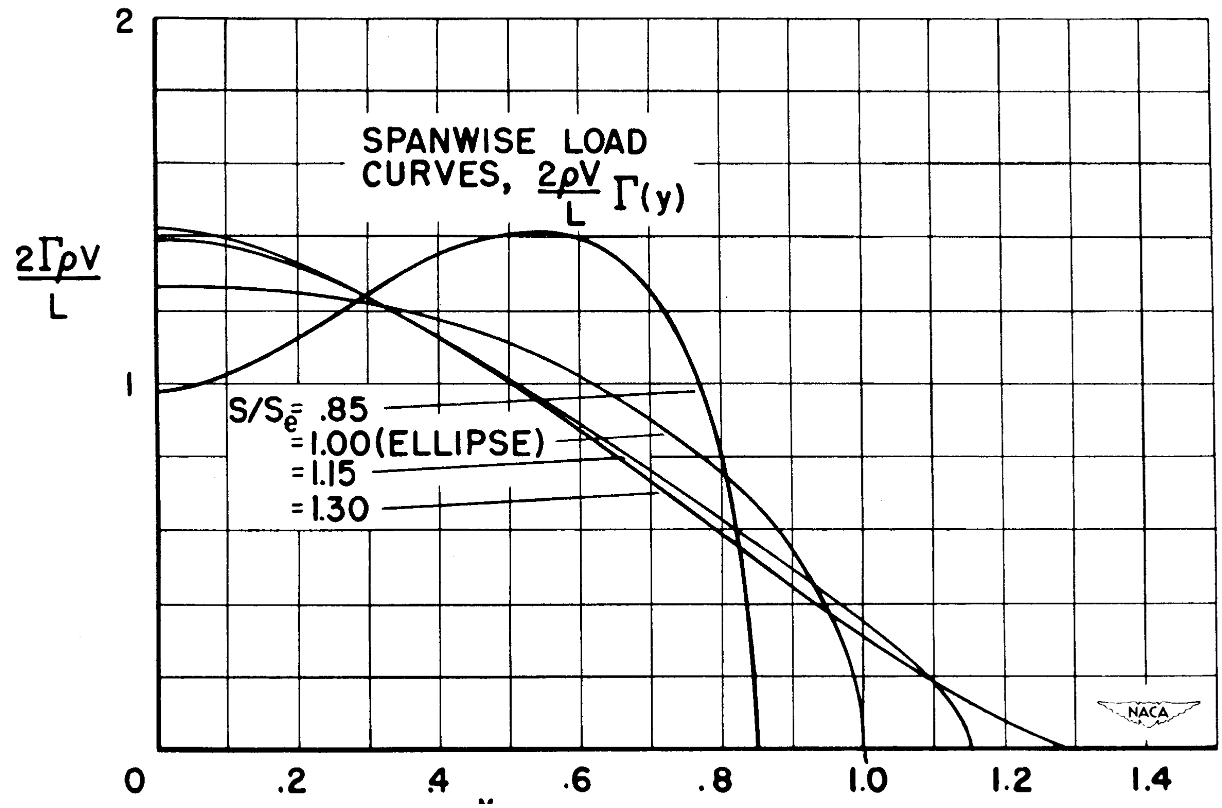
- Horten Spanload (1940-1955)
  induced thrust at tips
  wing root bending moment

Flight Mechanics Implications

- Proverse/Adverse induced yawing moments
  Force vectors on tips (twist & upwash)



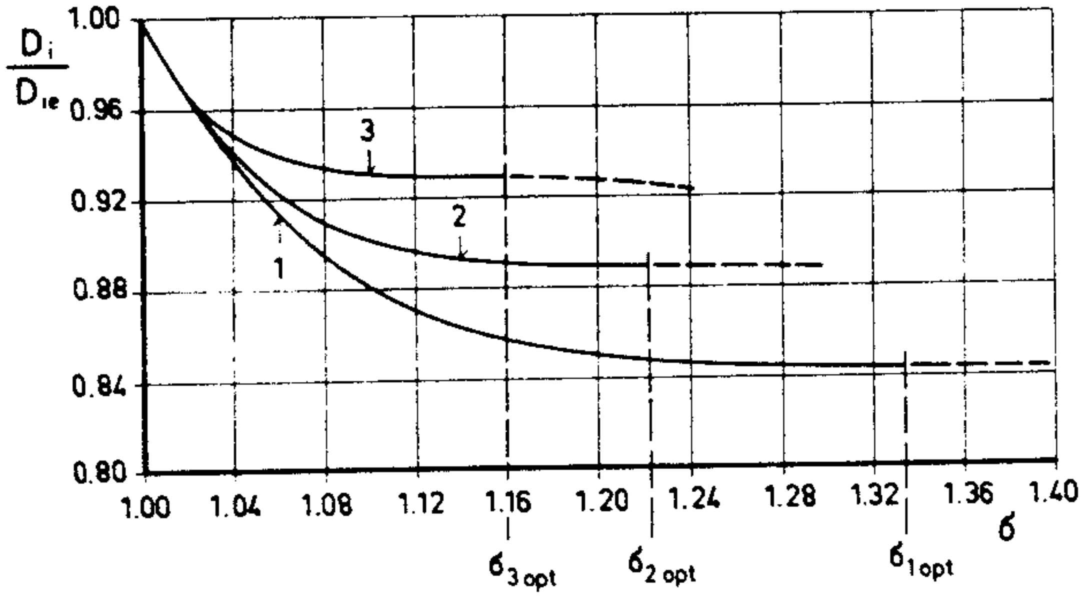
Jones Spanload



- Minimize induced drag (1950)
  Constrain wing root bending moment
  30% increase in span with 17% decrease in induced drag

- "Hence, for a minimum induced drag with a given total lift and a given bending moment the downwash must show a linear variation along the span." y = bx + c

Klein and Viswanathan



- Minimize induced drag (1975)
  Constrain bending moment
  Constrain shear stress
  16% increase in span with 7% decrease in induced drag

- "Hence the required downwash-distribution is parabolic." y = ax + bx + c

Spanload Summary

- Prandtl/Munk (1914)
  Elliptical
  Constrained only by span and lift
  Downwash: $y = c$

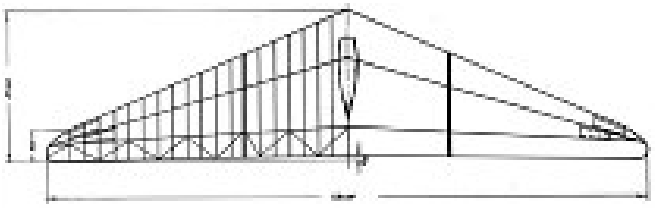
- Prandtl/Horten/Jones (1932)
  Bell shaped
  Constrained by lift and bending moment
  Downwash: $y = bx + c$

- Klein/Viswanathan (1975)
  Modified bell shape
  Constrained by lift, moment and shear (minimum structure)
  Downwash: $y = ax \quad + bx + c$

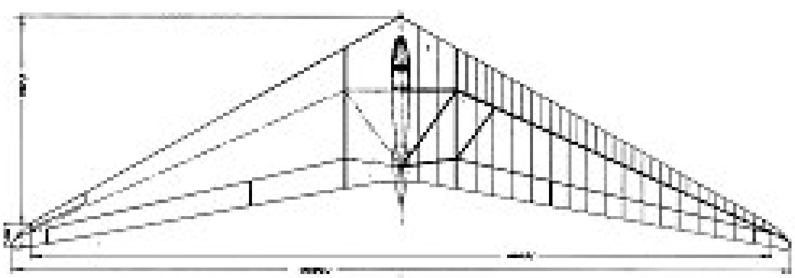
- Summarized by Jones (1979)

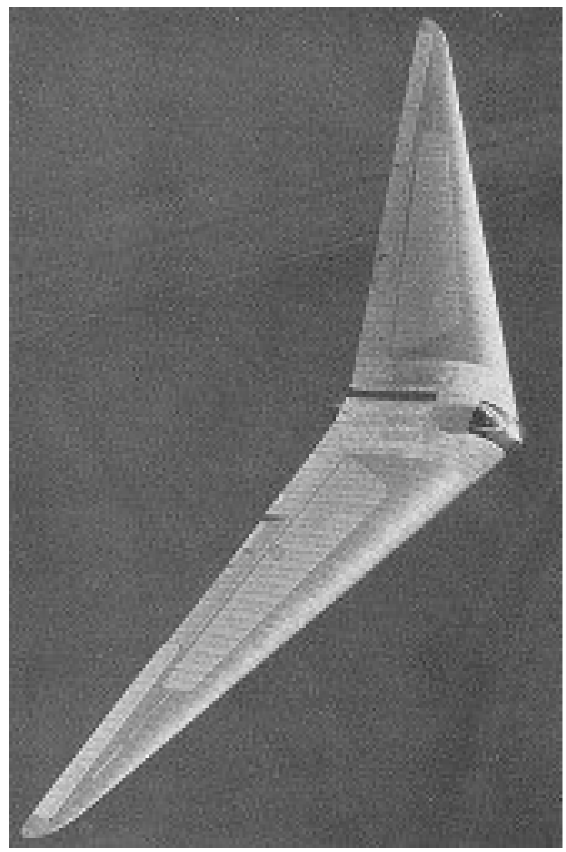
Early Horten Sailplanes (Germany)

- Horten I - 12m span

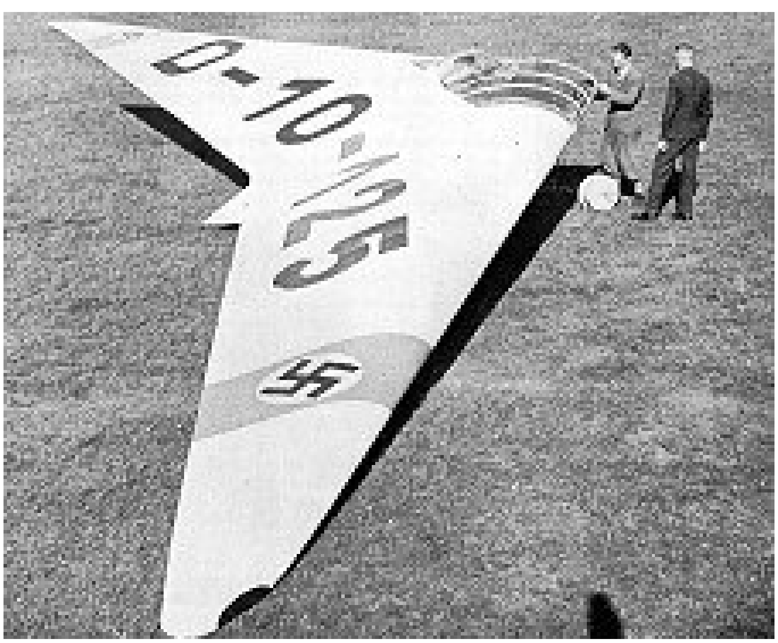


- Horten II - 16m span



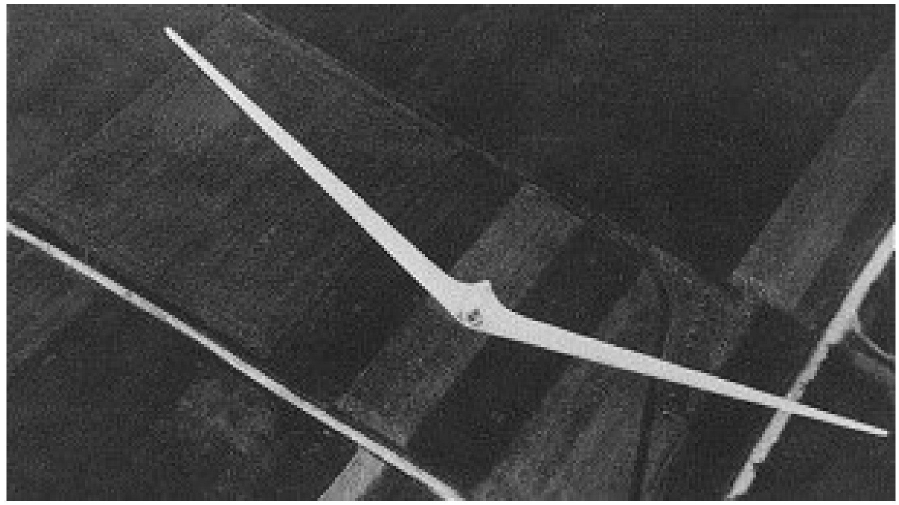


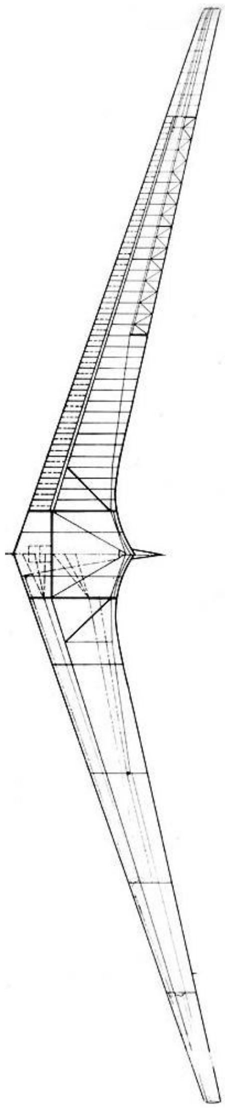
- Horten III - 20m span



Horten Sailplanes (Germany)

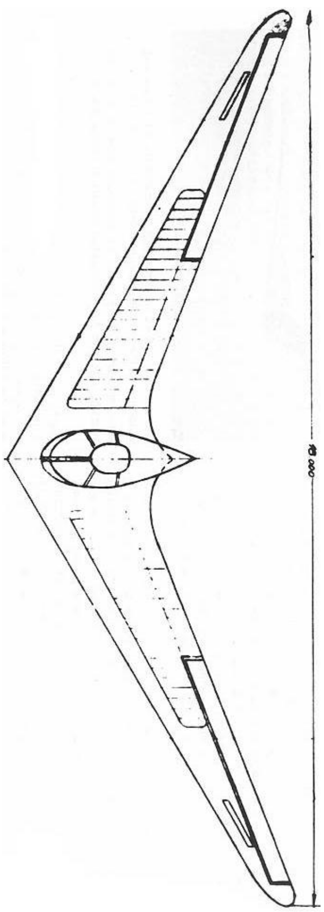
- H IV - 20m span
- H VI - 24m span

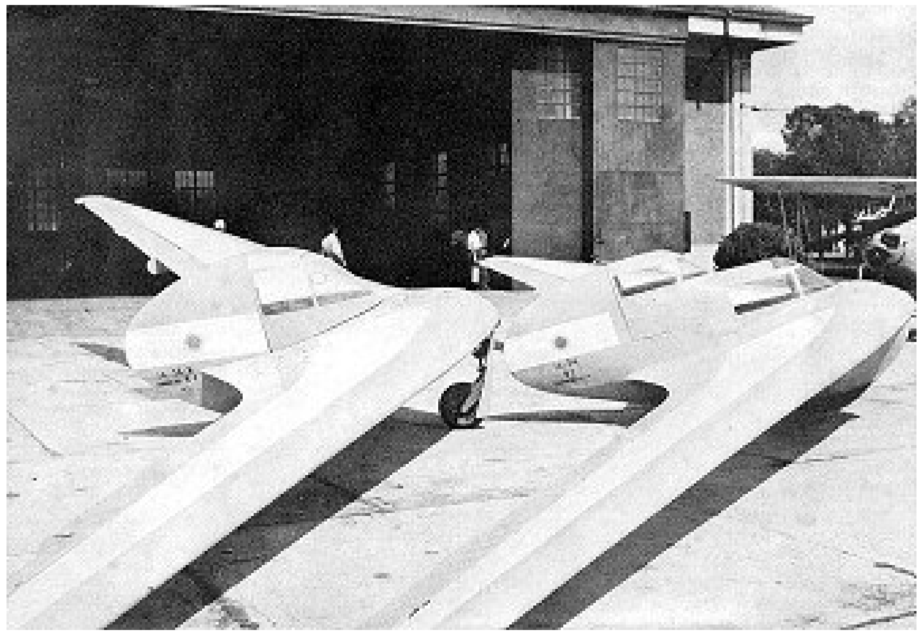




Horten Sailplanes (Argentina)

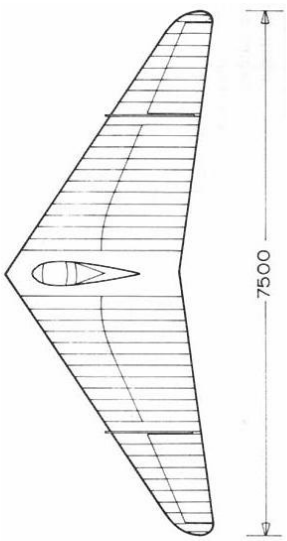
- H I b/c - 12m span
- H XV a/b/c - 18m span

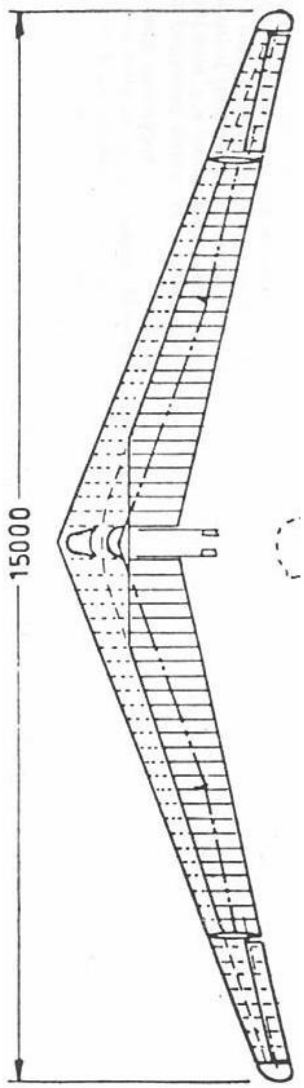


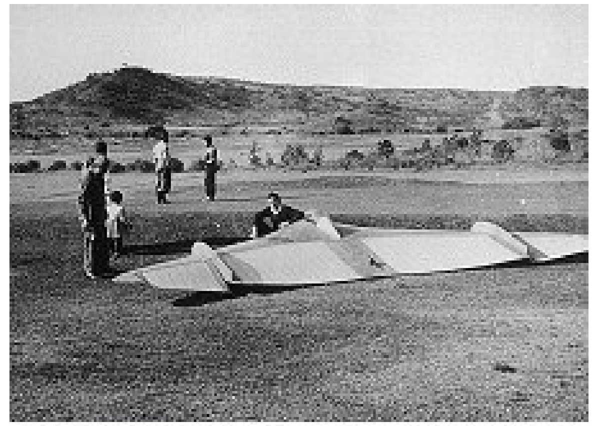


Later Horten Sailplanes (Argentina)

- H Xa/b/c 7.5m, 10m, & 15m





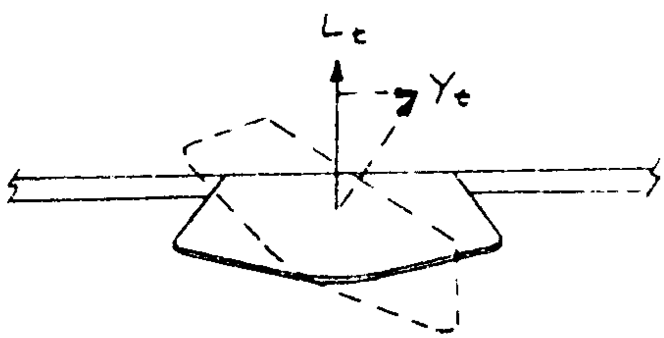


Bird Flight Model

- Minimum Structure
- Flight Mechanics Implications
- Empirical evidence

How do birds fly?

- Proverse/adverse yaw only solves constant turn rate problem
- Roll/yaw acceleration needed to initiate turns
- Need for a tail arises for maneuvering (“agility”)
- “First the tail is tilted downward on the side away from the direction of the turn…Perhaps the tail functions as a rudder in starting the turn…” (Koford, 1950)
- “…the tail was loaded upward and the same clockwise tail rotation produced a right force, thus a left turn…” (Hoey, 1992)



(b) UP-LOADED TAIL
(AFT CG)

Horten Spanload Equivalent to Birds

- Horten spanload is equivalent to bird span load (shear not considered in Horten designs)
- Flight mechanics are the same - turn components are the same
- Both attempt to use minimum structure
- Solve minimum drag, turn performance, and optimal structure with one solution

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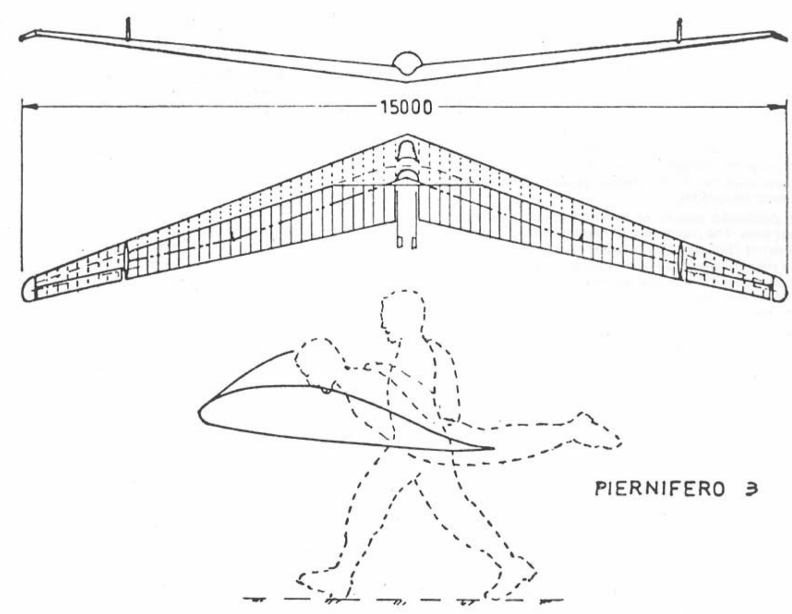
Horten H Xc Example

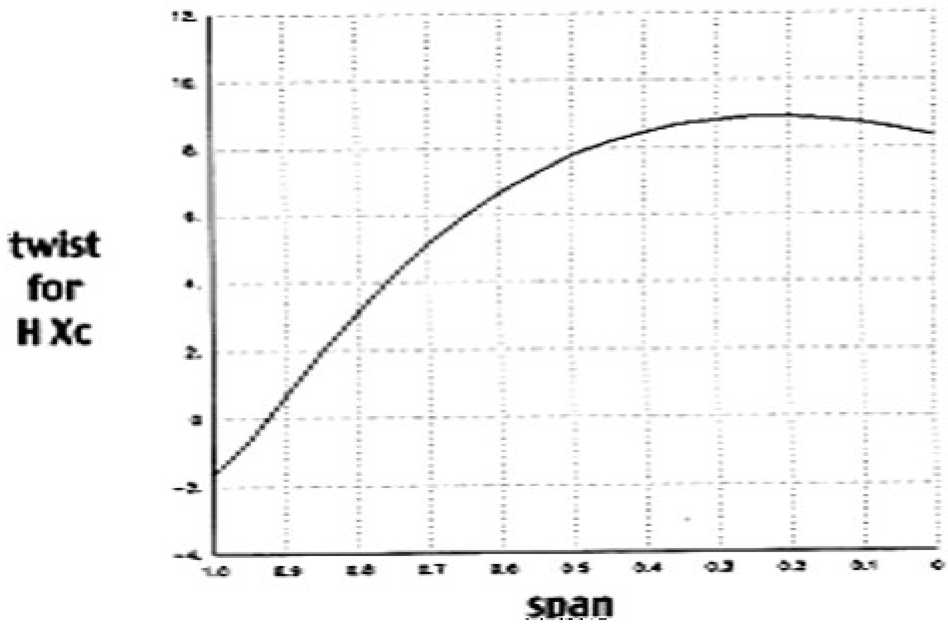
- Horten H Xc footlaunched ultralight sailplane 1950

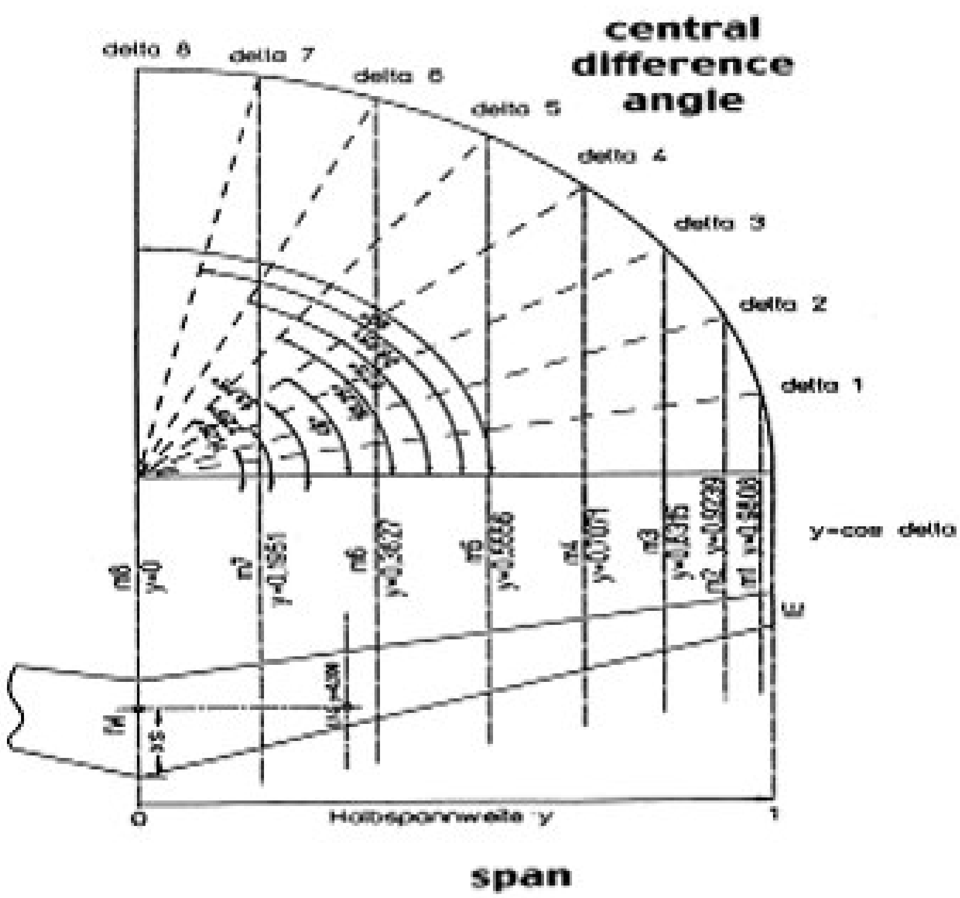


Skizze der H X c mit 15 Meter Spannweite. (Zeichnung Jan Scott)

Calculation Method

- Taper
- Twist
- Control Surface Deflections
- Central Difference Angle

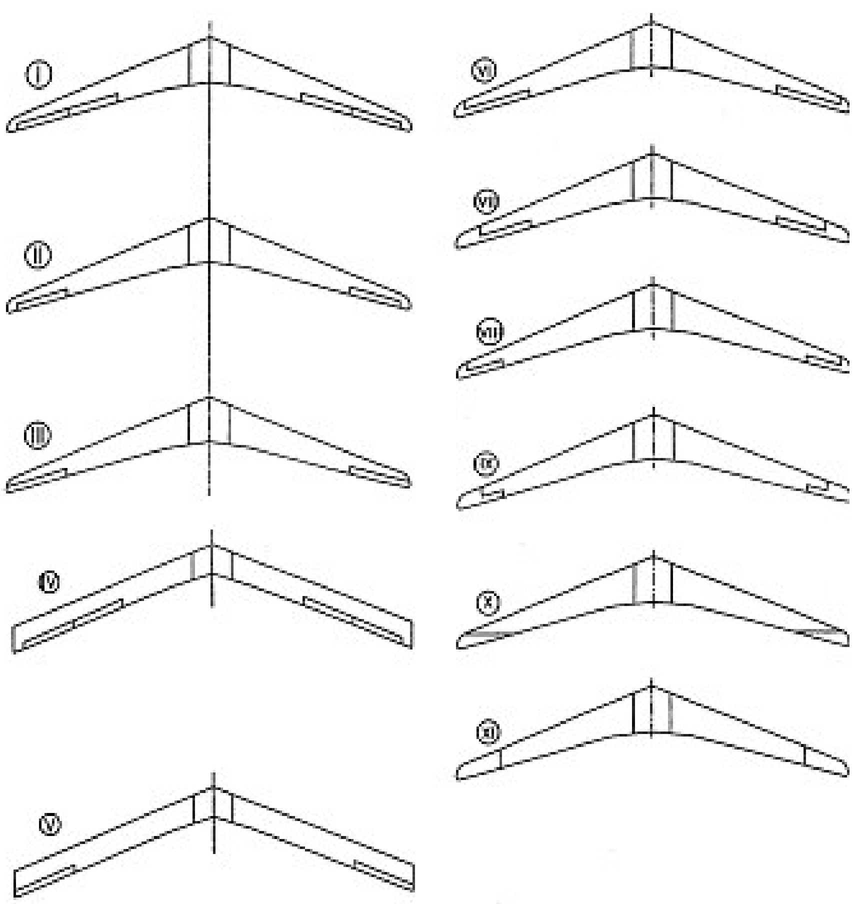




Dr Edward Udens' Results

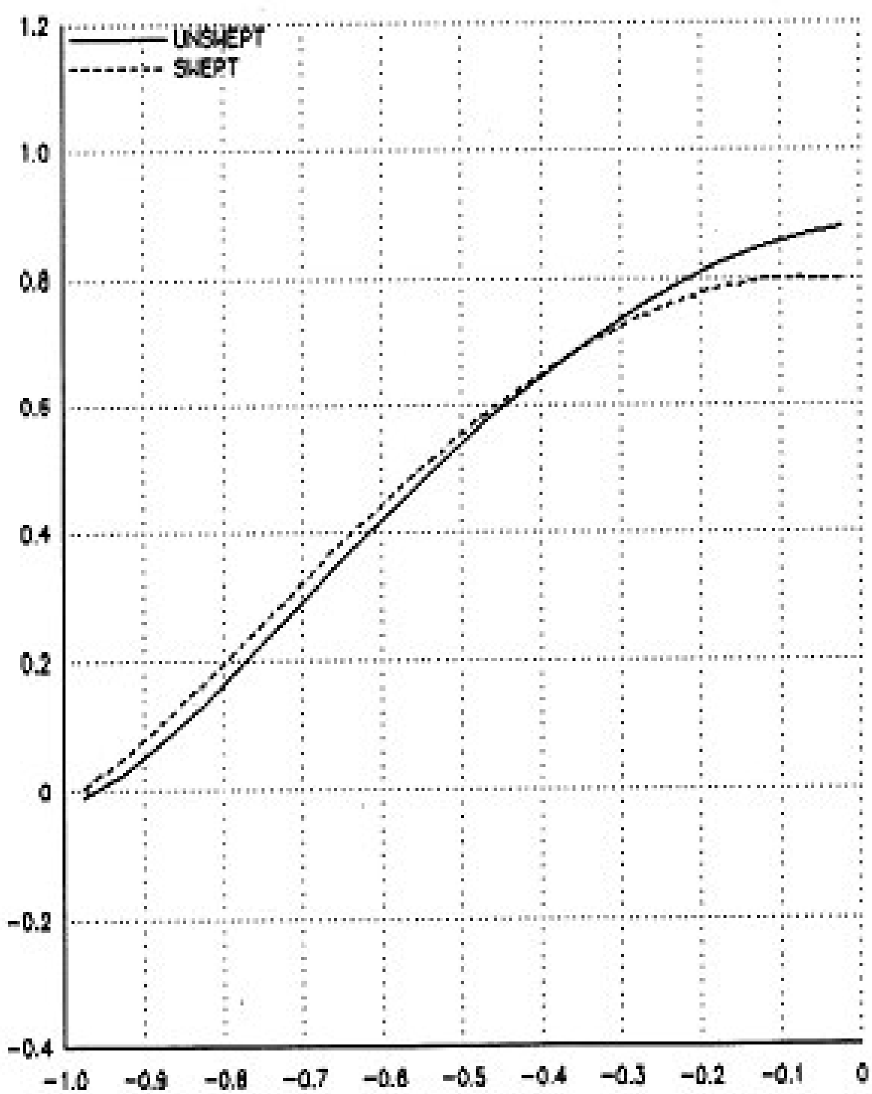
- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

Elevon Config	Cn∂a	Spanload
I	-.002070	bell
II	.001556	bell
III	.002788	bell
IV	-.019060	elliptical
V	-.015730	elliptical
VI	.001942	bell
VII	.002823	bell
VIII	.004529	bell
IX	.005408	bell
X	.004132	bell



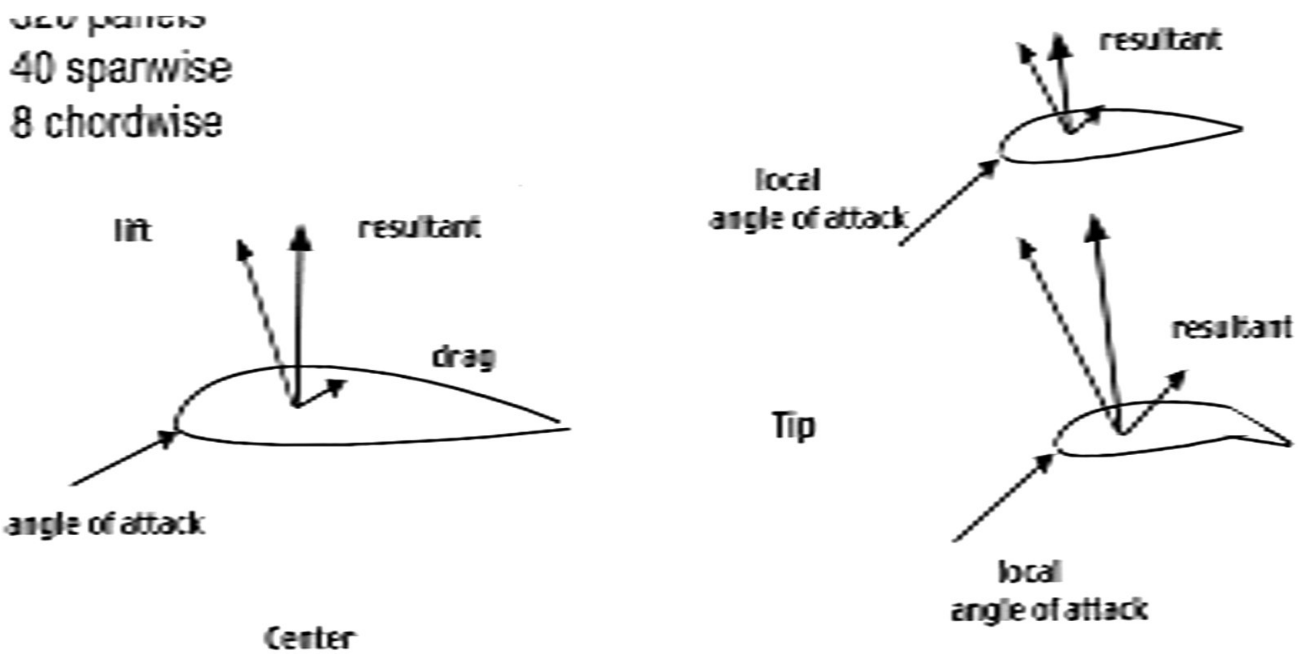
“Mitteleffekt”

- Artifact of spanload approximations
- Effect on spanloads
  increased load at tips
  decreased load near centerline
- Upwash due to sweep unaccounted for



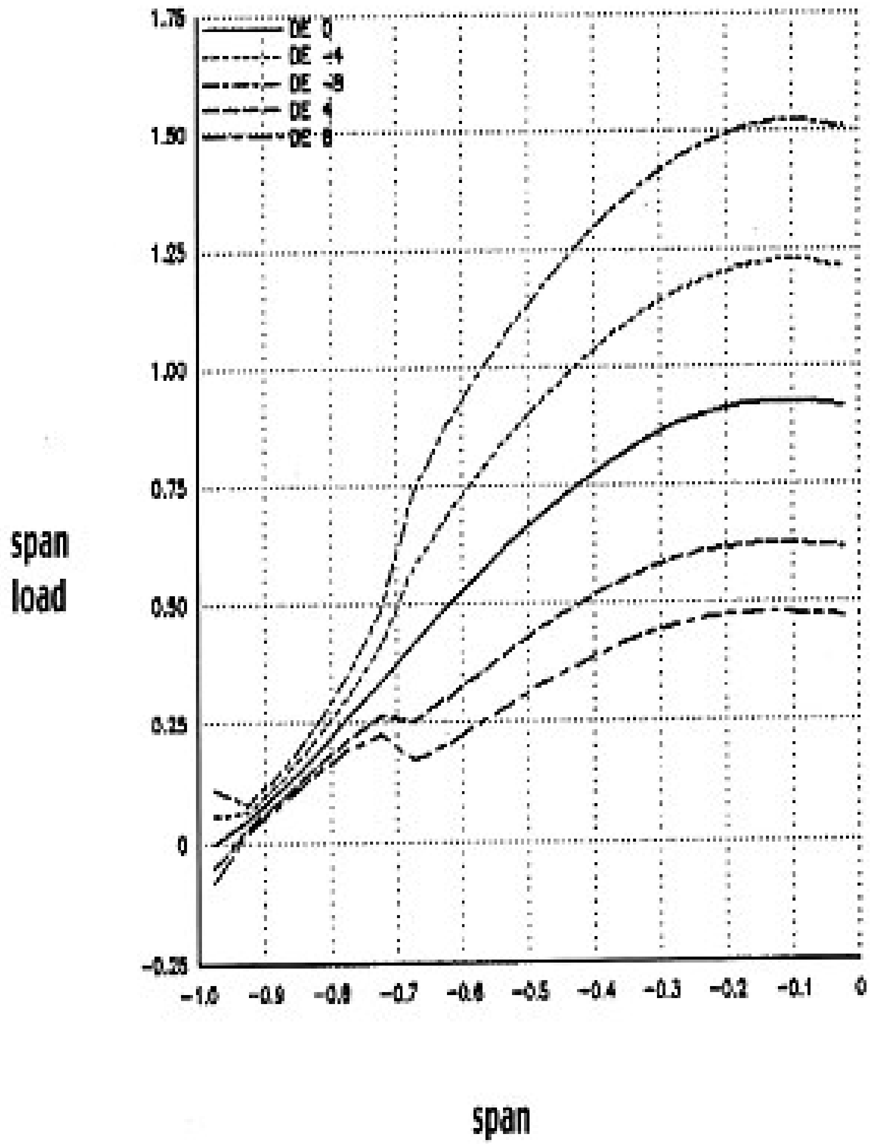
Horten H Xc Wing Analysis

- Vortex Lattice Analysis
- Spanloads (longitudinal & lateral-directional) - trim & asymmetrical roll
- Proverse/Adverse Induced Yawing Moments handling qualities
- Force Vectors on Tips - twist, elevon deflections, & upwash
- 320 Panels: 40 spanwise & 8 chordwise



Symmetrical Spanloads

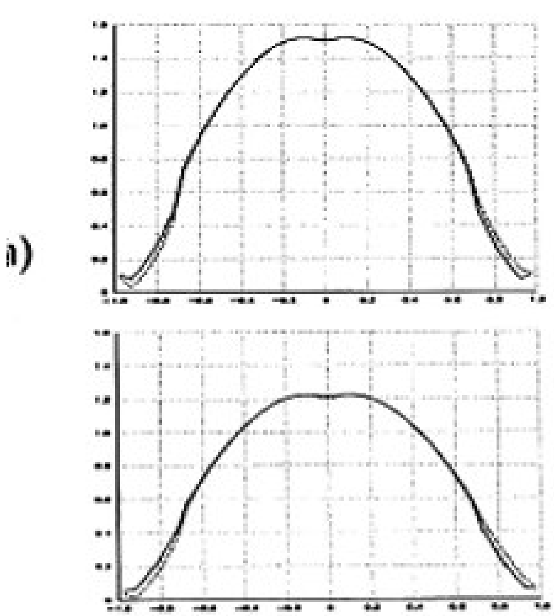
- Elevon Trim
- CG Location

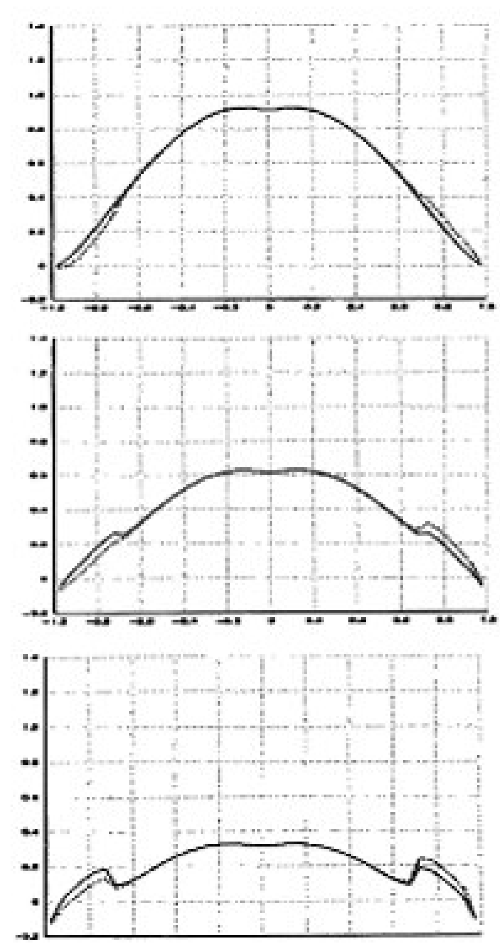


Asymmetrical Spanloads

- Cl∂a (roll due to aileron)
- Cn∂a (yaw due to aileron)
  induced component
  profile component
  change with lift
- Cn∂a/Cl∂a

- CL(Lift Coefficient)
  Increased lift:
   increased Clβ
  increased Cnβ*
  Decreased lift:
   decreased Clβ
  decreased Cnβ*





CL	Cl	Cn
.966	.01384	.00055
.774	.01384	.00037
.582	.01345	.00021
.390	.01384	.00003
.198	.01345	-.00015

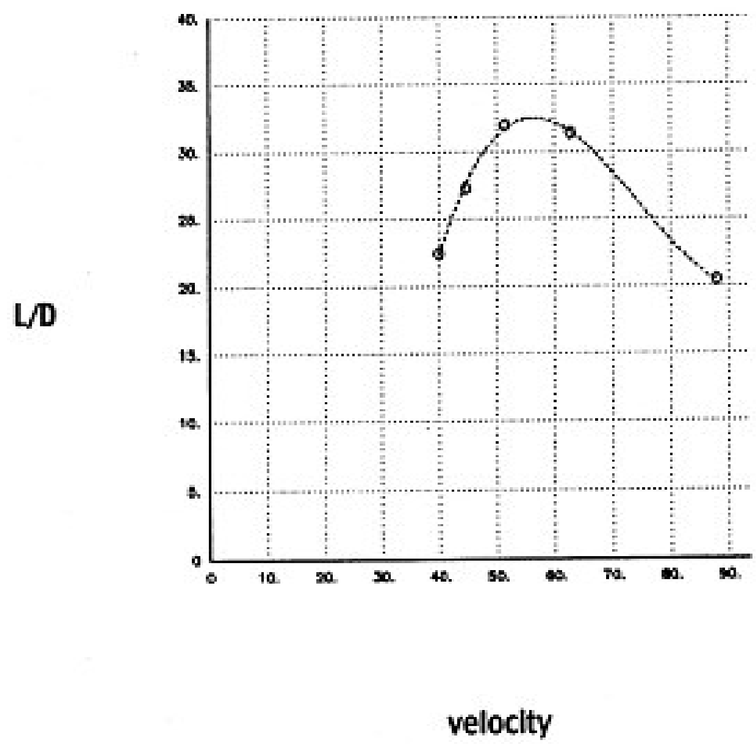
Airfoil and Wing Analysis

- Profile code (Dr Richard Eppler)
- Flap Option (elevon deflections)
- Matched Local Lift Coefficients
- Profile Drag
- Integrated Lift Coefficients
  match Profile results to Vortex Lattice
  separation differences in lift

Performance Comparison

- Max L/D: 31.9
- Min sink: 89.1 fpm
- Does not include pilot drag

- Prediicted L/D: 30
- Predicted sink: 90 fpm



Concluding Remarks

- Birds as the first model for flight
- Theoretical developments independent of applications
- Applied approach gave immediate solutions, departure from bird flight
- Eventual meeting of theory and applications (applied theory)
- Spanload evolution (Prandtl/Munk, Prandtl/Horten/Jones, Klein & Viswanathan)
- Flight mechanics implications
- Hortens are equivalent to birds
- Thanks: Walter Horten, Georgy Dez-Falvy, Bruce Carmichael, R.T. Jones, Russ Lee, Dan & Jan Armstrong, Dr Phil Burgers, Ed Lockhart, Andy Kecskes, Dr Paul MacCready, Reinhold Stadler, Edward Udens, & Jack Lambie

References

- Anderson, John Jr: “A History of Aerodynamics: and Its Impact on Flying Machines”; Cambridge University Press; Cambridge, United Kingdom.
- Prandtl, Ludwig: “Applications of Modern Hydrodynamics to Aeronautics”; NACA Report No. 116; 1921.
- Munk, Max M.: “The Minimum Induced Drag of Aerofoils”; NACA Report No. 121, 1923.
- Prandtl, Ludwig: ”Uber Tragflugel kleinsten induzierten Widerstandes”; Zeitschrift fur Flugtecknik und Motorluftschiffahrt, 28 XII 1932; Munchen, Deustchland.
- Horten, Reimar; and Selinger, Peter; with Scott, Jan (translator): “Nurflugel: the Story of Horten Flying Wings 1933 - 1960”; Weishapt Verlag; Graz, Austria; 1985.
- Horten, Reimar; unpublished personal notes.
- Udens, Edward; unpublished personal notes.
- Jones, Robert T.; “The Spanwise Distribution of Lift for Minimum Induced Drag of Wings Having a Given Lift and a Given Bending Moment”; NACA Technical Note 2249, Dec 1950.
- Klein, Armin and Viswanathan, Sathy; “Approximate Solution for Minimum induced Drag of Wings with a Given Structural Weight”; Journal of Aircraft, Feb 1975, Vol 12 No 2, AIAA.
- Whitcomb, R.T.; “A Design Approach and Selected Wind Tunnel Results at high Subsonic Speeds for Wing-Tip Mounted Winglets,” NASA TN D-8260, July 1976.
- Jones, Robert T; “Minimizing induced Drag.”; Soaring, October 1979, Soaring Society of America.
- Koford, Carl; “California Condor”; Audobon Special Report No 4, 1950, Dover, NY.
- Hoey, Robert; “Research on the Stability and Control of Soaring Birds”; AIAA Report 92-4122-CP, AIAA, 1992.